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Model independent search for signals of heavy Z' gauge boson in four-fermion processes is analyzed. It is shown that the renormalizability of the underlying theory formulated as a problem of scattering in an "external" field of heavy particle may be implemented in specific relations between various processes. Considering the two-Higgs-doublet model as the low-energy theory, two types of the Z' interactions with light particles are found to be compatible with renormalizability. They are distinguished as the Abelian and the "chiral" couplings. Observables proper to detect uniquely Z' in both cases are introduced.

I. INTRODUCTION

The existence of heavy Z' gauge boson is predicted by a number of grand unified theories (GUT's) and superstring theories [1]. The mass of this particle is expected to be of order $m_{Z'} \geq 500$ GeV, and therefore it cannot be produced at present day accelerators. Various strategies of searching for signals of Z' as the virtual heavy state are developed and different observables giving possibility of its experimental detection have been proposed (see survey [2] and references therein). The model-dependent and model-independent Z' search at e^+e^- colliders is widely discussed (see, for instance, report [3]). A popular model assumes that at low energies the Z' interactions with ordinary particles of the Standard Model (SM) can be described by the effective gauge group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \tilde{\mathrm{U}}(1)$. An alternative choice is the gauge group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ [2,4]. These models are considered as the remnants of underlying theories which are not specified. The low-energy effective Lagrangians (EL) take into consideration the most general property that is inherent the renormalizable theories and ensured by the decoupling theorem [5] – the dominance of renormalizable interactions at low energies. The interactions of non-renormalizable types, being generated at high energies due to radiation corrections, are suppressed by the inverse heavy mass $1/m_{Z'}$ and therefore can be skipped in lower order. Another popular description is the construction of the EL considered as the sum of all effective operators with dimensions $n \geq 4$ which are formed from the fields of light particles. The coefficients at these operators are treated as independent unknown numbers to be determined in experiments. For more details see Ref. [6]. In general, the number of the Z' couplings is large. So, it is difficult to introduce observables allowing the unique detection of the Z' signals. In this regard, it is desirable either to decrease the number of the independent Z' parameters on some reasonable grounds and to introduce observables most sensitive to the Z' virtual states. In any case, the main idea is to find correlations between the Z' couplings at low energies.

A straightforward way to find the correlations is to specify an underlying theory describing interactions at energies $\sim \Lambda_{\mathrm{GUT}}$ and to consider running of the couplings from high to low energies $\sim m_W$ by using the renormalization group (RG) equations. In this approach each underlying theory leads to the unique values of the parameters and, hence, the corresponding correlations are the model dependent ones. Another way is to specify a basis low-energy theory (for instance, the SM can be chosen) and to determine the relations between the Z' parameters which follow from model independent arguments. These correlations are to be model independent. Naturally, they remain dependent on the basis low-energy theory.

In Refs. [7,8] a method of derivation of the model independent correlations between the parameters of physics beyond the SM has been developed and new observables which are useful in searching for Z' boson in four-fermion processes were proposed. That is based on principles of the RG and the decoupling theorem [9]. As it was argued, a virtual heavy particle can be treated as an "external field" scattering the SM particles. The factor that enters the vertex describing interaction with the field is model dependent and considered as an arbitrary parameter. Because of renormalizability, the scattering amplitude in the field satisfies a simple relation (named the RG relation) which includes the β and γ functions, calculated with the light particles only, and the vertex factor. Hence, relations between different vertex factors follow. Then, they can be implemented in a number of model independent observables proper to the specific heavy virtual state, in particular, for Z' particle [8].

In Ref. [7] the case of the Abelian Z' beyond the minimal SM (with one scalar doublet) has been investigated. However, at present there is a few information about scalar fields of the SM. In this regard, the theory with two scalar doublets is intensively studied [10,11]. The two-Higgs-doublet model (THDM) is also known as the low-energy limit of some E_6 based GUT's which predict the Z' boson. In the present paper the results of Ref. [7]

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are generalized to the THDM case. We analyse in detail both the Abelian and the so called "chiral" types of the Z' couplings to light particles. As the latter type, we first assume the most general parametrization of the Z' interactions with the SM fields and then derive the structure of the generators which is compatible with renormalizability. As it will be shown, there is important difference between these two cases.

Thus, in order to derive model independent constraints, we choose the THDM as the low-energy basis theory (notice, the minimal SM is the particular case of the THDM). Then, we introduce the general gauge parametrization of linear in the Z' couplings which is independent of the specific underlying theory. As a result, the derived RG correlations are the model independent ones. They hold for the THDM as well as for the minimal SM. Moreover, the existence of other heavy particles with masses $m_i \geq m_{Z'}$ does not affect them.

As it will be shown, there are two completely different sets of the Z' couplings to the SM fields compatible with renormalizability. The first one describes the Abelian Z' which respects the additional $\tilde{\mathrm{U}}(1)$ symmetry of the low energy EL. In this case, the Z' couplings to the axial-vector fermion currents have the universal absolute value. The second set corresponds to the chiral Z' which interacts with the SM doublets, only. One has to distinguish these neutral Z' gauge bosons since they are described by different operators.

The content is as follows. In Sec. II the general parametrization of interactions involving the Z' and the SM fields is introduced. The RG correlations between the Z' couplings are derived in Sec. III. In Sec. IV they are checked for the specific values of the Z' couplings in the GUT's based on the E_6 group. In Sec. V observables allowing the detection of the Z' signal are proposed. The results of the investigation are discussed in Sec. VI.

II. PARAMETRIZATION OF THE Z' COUPLINGS

In the present paper we are going to analyze four-fermion scattering amplitudes of order $\sim m_{Z'}^{-2}$ generated by the virtual Z' state. Interactions involving more than one Z' field contribute to amplitudes with several Z' states, which are of order $m_{Z'}^{-4}$ and higher. Therefore, in carrying out calculations we are interested in linear in the Z' vertices, only.

In order to introduce the general parametrization of the vertices involving SM fields and linear in the Z' state, one has to impose a number of natural conditions. First of all, the Z' interactions of renormalizable types are dominant at low energies $\sim m_W$. The non-renormalizable interactions generated at high energies due to radiation corrections are suppressed by the inverse heavy scales $1/\Lambda_i$ and therefore can be neglected. We assume that the Z' is the only neutral vector boson with the mass $\sim m_{Z'}$, and the Z' gauge field enters the theory through covariant derivatives with the corresponding charge. We also assume that the SM gauge group, $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, is the subgroup of the GUT group. In this case a product of generators associated with the SM subgroup is a linear combination of these generators. As a consequence, all the structure constants connecting two SM gauge bosons with Z' are to be zero. Therefore, the interactions of gauge fields of the types $Z'W^+W^-$, $Z'ZZ$ and other are absent at tree level.

Let ϕ_i $(i = 1, 2)$ denotes two complex scalar doublets:

$$\phi_i = \left\{ a_i^+, \frac{v_i + b_i + ic_i}{\sqrt{2}} \right\}, \qquad (1)$$

where v_i marks the corresponding vacuum expectation values, a_i^+ are complex fields, and b_i, c_i are real fields. By diagonalizing the quadratic terms of the scalar potential $V(\phi_1, \phi_2)$ one obtains the mass eigenstates: two neutral CP-even scalar particles, H and h, a neutral CP-odd scalar particle, A_0, the Goldstone boson partner of the Z boson, χ_3, the charged Higgs field, $H^\pm$, and the Goldstone field associated with the $W^\pm$ boson, $\chi^\pm$:

$$\begin{aligned}
a_1^+ &= \chi^+ \cos\beta - H^+ \sin\beta, & a_2^+ &= H^+ \cos\beta + \chi^+ \sin\beta, \\
c_1 &= \chi_3 \cos\beta - A_0 \sin\beta, & c_2 &= A_0 \cos\beta + \chi_3 \sin\beta, \\
b_1 &= H \cos\alpha - h \sin\alpha, & b_2 &= h \cos\alpha + H \sin\alpha, \qquad (2)
\end{aligned}$$

where

$$\tan\beta = \frac{v_1}{v_2}, \qquad (3)$$

and the angle α is determined by the explicit form of the potential $V(\phi_1, \phi_2)$. For instance, the CP-conserving potential which has only CP-invariant minima can be used [10,11]:

$$\begin{aligned}
V = &\sum_{i=1}^{2} \left[-\mu_i^2 \phi_i^\dagger \phi_i + \lambda_i (\phi_i^\dagger \phi_i)^2 \right] + \lambda_3 (\mathrm{Re}[\phi_1^\dagger \phi_2])^2 \\
&+ \lambda_4 (\mathrm{Im}[\phi_1^\dagger \phi_2])^2 + \lambda_5 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2). \qquad (4)
\end{aligned}$$

It is consistent with absence of the tree-level flavor-changing neutral currents (FCNC) in the fermion sector. The corresponding value of α is [11]

$$\tan 2\alpha = -\frac{v_1 v_2 (\lambda_3 + \lambda_5)}{\lambda_2 v_2^2 - \lambda_1 v_1^2}. \qquad (5)$$

At low energies when all heavy states are decoupled the Z' interactions with the scalar doublets can be parametrized in a model independent way as follows [2]:

$$\begin{aligned}
\mathcal{L}_\phi = \sum_{i=1}^{2} \Bigg| \Bigg(&\partial_\mu - \frac{ig}{2} \sigma_a W_\mu^a - \frac{ig'}{2} Y_{\phi_i} B_\mu \\
&- \frac{i\tilde{g}}{2} \tilde{Y}_{\phi_i} \tilde{B}_\mu \Bigg) \phi_i \Bigg|^2, \qquad (6)
\end{aligned}$$

where g, g', $\tilde{g}$ are the charges associated with the $\mathrm{SU}(2)_L$, $\mathrm{U}(1)_Y$, and the Z' gauge groups, respectively, σ_a are the Pauli matrices,

$$\tilde{Y}_{\phi_i} = \begin{pmatrix} \tilde{Y}_{\phi_i,1} & 0 \\ 0 & \tilde{Y}_{\phi_i,2} \end{pmatrix} \tag{7}$$

is the generator corresponding to the Z' boson gauge group, and Y_{ϕ_i} is the $\mathrm{U}(1)_Y$ hypercharge. The condition $Y_{\phi_i} = 1$ guarantees that the vacuum is invariant with respect to the photon gauge group.

The physical vector bosons, A, Z, and Z', are related to the symmetry eigenstates as follows:

$$\begin{aligned} B &\to A\cos\theta_W - (Z\cos\theta_0 - Z'\sin\theta_0)\sin\theta_W, \\ W_3 &\to A\sin\theta_W + (Z\cos\theta_0 - Z'\sin\theta_0)\cos\theta_W, \\ \tilde{B} &\to Z\sin\theta_0 + Z'\cos\theta_0, \end{aligned} \tag{8}$$

where $\tan\theta_W = g'/g$ is the SM value of the Weinberg angle, and

$$\tan\theta_0 = \frac{\tilde{g} m_W^2 \left(\tilde{Y}_{\phi_1,2}\cos^2\beta + \tilde{Y}_{\phi_2,2}\sin^2\beta\right)}{g\cos\theta_W\left(m_{Z'}^2 - m_W^2/\cos^2\theta_W\right)}. \tag{9}$$

As is seen, the mixing angle θ_0 is of order $\sim m_W^2/m_{Z'}^2$. That results in the corrections $\sim m_W^2/m_{Z'}^2$ to the interactions between the SM particles. To avoid tree-level mixing between the physical Z boson and the physical scalar field A_0 one has to impose the condition $\tilde{Y}_{\phi_1,2} = \tilde{Y}_{\phi_2,2} \equiv \tilde{Y}_{\phi,2}$.

We also introduce the general parametrization of the fermion-vector Lagrangian [2,4,12]:

$$\begin{aligned} \mathcal{L}_f = {} & i\sum_{f_L} \bar{f}_L\gamma^\mu\Big(\partial_\mu - \frac{ig}{2}\sigma_a W_\mu^a - \frac{ig'}{2}B_\mu Y_{f_L} \\ & - \frac{i\tilde{g}}{2}\tilde{B}_\mu \tilde{Y}_{f_L}\Big) f_L \\ & + i\sum_{f_R} \bar{f}_R\gamma^\mu\Big(\partial_\mu - ig'B_\mu Q_f - \frac{i\tilde{g}}{2}\tilde{B}_\mu\tilde{Y}_{R,f}\Big) f_R, \end{aligned} \tag{10}$$

where the summation over all the SM left-handed fermion doublets, $f_L = \{(f_u)_L, (f_d)_L\}$, and the right-handed singlets, $f_R = (f_u)_R, (f_d)_R$, is understood, Q_f denotes the charge of f in the positron charge units,

$$\tilde{Y}_{f_L} = \begin{pmatrix} \tilde{Y}_{L,f_u} & 0 \\ 0 & \tilde{Y}_{L,f_d} \end{pmatrix}, \tag{11}$$

and Y_{f_L} equals to -1 for leptons and $1/3$ for quarks.

Renormalizable interactions of the fermions and the scalars are described by the Yukawa Lagrangian. To avoid the existence of the tree-level FCNC one has to ensure that the diagonalization of the fermion mass matrix automatically diagonalizes the scalar-fermion couplings. In this case the Yukawa Lagrangian which respects the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge group can be written in the form:

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} = {} & -\sqrt{2}\sum_{f_L}\sum_{i=1}^{2}\Big\{G_{f_d,i}\left[\bar{f}_L\phi_i(f_d)_R + (\bar{f}_d)_R\phi_i^\dagger f_L\right] \\ & + G_{f_u,i}\left[\bar{f}_L\phi_i^c(f_u)_R + (\bar{f}_u)_R\phi_i^{c\dagger} f_L\right]\Big\}, \end{aligned} \tag{12}$$

where $\phi_i^c = i\sigma_2\phi_i^*$ is the charge conjugated scalar doublet, and the Cabibbo-Kobayashi-Maskawa mixing is neglected. Then, the masses of the fermions are

$$m_f = \frac{2m_W}{g}\left(G_{f,1}\cos\beta + G_{f,2}\sin\beta\right). \tag{13}$$

As have been shown by Glashow and Weinberg [13], the tree-level FCNC mediated by Higgs bosons are absent if all fermions of a given electric charge couple to no more than one Higgs doublet. This restriction leads to four different models as discussed in Ref. [11]. In what follows we will use the most general parametrization (12) which includes the models mentioned as well as other possible variations of the Yukawa sector without tree-level FCNC.

By employing Eqs. (6), (10), and (12) it is easy to derive the Feynman rules which are collected in Appendix A.

III. RG RELATIONS

Now, we are going to consider the correlations between the parameters $\tilde{Y}_{L,f}$, $\tilde{Y}_{R,f}$, $\tilde{Y}_{\phi_i,1}$, and $\tilde{Y}_{\phi_i,2}$ due to renormalizability of the underlying theory.

As is known, the S-matrix elements are to be invariant with respect to the RG transformations which express the independence of the normalization point κ. In a theory with different mass scales the decoupling of heavy loop contributions at the thresholds of heavy masses, Λ, results in the important property of the low energy amplitudes: the running of the proper functions is regulated by the loops of light particles. Therefore, the β and γ functions at low energies are determined by the SM particles, only. This fact is the consequence of the decoupling theorem [9].

Actually, the decoupling results in the redefinition of the parameters of the theory which removes all the heavy particle loop contributions proportional to $\ln\kappa$ from the RG equation [5,14],

$$\begin{aligned} \lambda_a &= \hat{\lambda}_a + a_{\lambda_a}\ln\frac{\hat{\Lambda}^2}{\kappa^2} + b_{\lambda_a}\ln^2\frac{\hat{\Lambda}^2}{\kappa^2} + ..., \\ X &= \hat{X}\left(1 + a_X\ln\frac{\hat{\Lambda}^2}{\kappa^2} + b_X\ln^2\frac{\hat{\Lambda}^2}{\kappa^2} + ...\right), \end{aligned} \tag{14}$$

where we use the notation λ_a to refer to the charges of the theory, and X represents all the fields and the masses. Hats over quantities mark the parameters of the underlying theory. They include the contributions of loops containing both the SM particles and the heavy ones, whereas the quantities without hats are calculated in the assumption that no heavy particles are excited inside loops. The matching between the both sets of parameters (λ_a, X and $\hat{\lambda}_a$, $\hat{X}$) is chosen to be done at the normalization point $\kappa \sim \Lambda$,

$$\lambda_a \mid_{\kappa=\Lambda} = \hat{\lambda}_a \mid_{\kappa=\Lambda}, \quad X \mid_{\kappa=\Lambda} = \hat{X} \mid_{\kappa=\Lambda} . \tag{15}$$

Since the parameters λ_a, X and $\hat{\lambda}_a$, $\hat{X}$ differ at the one-loop level, one can substitute one set of them by another in the one-loop RG equation.

As it was shown in Ref. [7], the redefinition of the fields and the charges (14) allows one to eliminate the one-loop mixing between heavy and light virtual states. Therefore, virtual states of heavy particles can be treated as the "external" fields scattering SM particles. The renormalizability of the underlying theory leads to the so called RG relations for the vertices describing this scattering.

Let us consider four-fermion amplitudes caused by the Z' boson exchange. In lower order in $m_W^2/m_{Z'}^2$ the process $\bar{f}_1 f_1 \to Z'^* \to \bar{f}_2 f_2$ can be presented as scattering of the initial, f_1, and the final, f_2, fermions in the "external" field $1/m_{Z'}$ with the corresponding vertex factors $\Gamma_{f_1 Z'}$, $\Gamma_{f_2 Z'}$. The quantity $\Gamma_{f Z'}$ contains no contributions of heavy particle loops. Thus, it can be computed as the linear combination of the parameters $\tilde{Y}_{L,f}$, $\tilde{Y}_{R,f}$, $\tilde{Y}_{\phi_i,1}$, and $\tilde{Y}_{\phi_i,2}$.

The RG invariance of the vertex means

$$\mathcal{D}\left(\bar{f}\Gamma_{f Z'} f \frac{1}{m_{Z'}}\right) = 0, \tag{16}$$

where the effective low-energy RG operator [5] is defined as follows:

$$\mathcal{D} \equiv \frac{\partial}{\partial \ln \kappa} + \sum_a \beta_a \frac{\partial}{\partial \lambda_a} - \sum_X \gamma_X \frac{\partial}{\partial \ln X},$$
$$\beta_a = \frac{d\lambda_a}{d \ln \kappa}, \quad \gamma_X = -\frac{d \ln X}{d \ln \kappa}, \tag{17}$$

where the coefficient functions β_a and γ_X are computed taking into account the loops of light particles.

The relation (16) ensures the leading logarithmic terms of the higher-loop vertex to reproduce the appropriate tree-level structure that is the consequence of renormalizability. The familiar usage of Eq. (16) is to improve scattering amplitudes calculated in a fixed order of perturbation theory. In contrast, in what follows we will apply Eq. (16) to obtain an algebraic relation between the parameters $\tilde{Y}_{L,f}$, $\tilde{Y}_{R,f}$, $\tilde{Y}_{\phi_i,1}$, $\tilde{Y}_{\phi_i,2}$, which are to be considered as arbitrary numbers, since the underlying theory is not specified. The reasons for that are as follow. In case when the underlying theory is specified ($\tilde{Y}_{L,f}$, $\tilde{Y}_{R,f}$, $\tilde{Y}_{\phi_i,1}$, $\tilde{Y}_{\phi_i,2}$ are to be computed as discussed before), and the β and γ functions as well as the S-matrix elements are calculated in a fixed order of perturbation theory, Eq. (16) is just the identity. If the underlying theory is not specified, whereas the β, γ functions and the S-matrix elements are computed in a fixed order of perturbation theory, the equality (16) may correlate the unknown parameters $\tilde{Y}$. In case of the four-fermion processes mediated by the gauge Z' boson, the number of independent β functions is less than the number of RG equations. Thus, the non-trivial system of equations correlating the originally independent parameters occurs.

The one-loop RG relation for the fermion-Z' vertex is [7]

$$\bar{f}\frac{\partial \Gamma^{(1)}_{f Z'}}{\partial \ln \kappa} f \frac{1}{m_{Z'}} + \mathcal{D}^{(1)}\left(\bar{f}\Gamma^{(0)}_{f Z'} f \frac{1}{m_{Z'}}\right) = 0, \tag{18}$$

where $\Gamma^{(0)}_{f Z'}$ and $\Gamma^{(1)}_{f Z'}$ denote the tree-level and the one-loop level contributions to the fermion-Z' vertex, and $\mathcal{D}^{(1)}$ is the one-loop level part of the RG operator:

$$\mathcal{D}^{(1)} \equiv \sum_a \beta^{(1)}_a \frac{\partial}{\partial \lambda_a} - \sum_X \gamma^{(1)}_X \frac{\partial}{\partial \ln X}. \tag{19}$$

As it follows from Eq. (18), only the divergent parts of the one-loop vertices $\Gamma^{(1)}_{f Z'}$ are to be calculated. The corresponding diagrams are shown in Fig. 1. The following expressions for the left-handed and the right-handed fermions, respectively, have been obtained,

$$\begin{aligned}
\frac{\partial \Gamma^{\mu}_{f_R Z'}}{\partial \ln \kappa} &= \frac{\gamma^\mu}{8\pi^2}\Big\{ g^2 Q_f^2 \tilde{Y}_{R,f} \tan^2\theta_W + \frac{4}{3} g^2_{s,f} \tilde{Y}_{R,f} \\
&\quad + G^2_{f,1}\left[2T^3_f\left(\tilde{Y}_{\phi,2} + \tilde{Y}_{\phi_1,1}\right) + \tilde{Y}_{L,f} + \tilde{Y}_{L,f^\star}\right] \\
&\quad + G^2_{f,2}\left[2T^3_f\left(\tilde{Y}_{\phi,2} + \tilde{Y}_{\phi_2,1}\right) + \tilde{Y}_{L,f} + \tilde{Y}_{L,f^\star}\right] \\
&\quad + O\left(\frac{m_W^2}{m_{Z'}^2}\right)\Big\}, \\
\frac{\partial \Gamma^{\mu}_{f_L Z'}}{\partial \ln \kappa} &= \frac{\gamma^\mu}{8\pi^2}\Big\{ \frac{g^2}{2} \tilde{Y}_{L,f^\star} + \frac{4}{3} g^2_{s,f} \tilde{Y}_{L,f} \\
&\quad + g^2 \tilde{Y}_{L,f}\left[\frac{1}{4\cos^2\theta_W} + \left(Q_f^2 - |Q_f|\right)\tan^2\theta_W\right] \\
&\quad + \left(G^2_{f,1} + G^2_{f,2}\right)\left(\tilde{Y}_{R,f} - 2T^3_f \tilde{Y}_{\phi,2}\right) \\
&\quad + G^2_{f^\star,1}\left(2T^3_f \tilde{Y}_{\phi_1,1} + \tilde{Y}_{R,f^\star}\right) \\
&\quad + G^2_{f^\star,2}\left(2T^3_f \tilde{Y}_{\phi_2,1} + \tilde{Y}_{R,f^\star}\right) \\
&\quad + O\left(\frac{m_W^2}{m_{Z'}^2}\right)\Big\},
\end{aligned} \tag{20}$$

where f and $f^\star$ are the partners of a $\mathrm{SU}(2)_L$ fermion doublet (namely, $l^\star = \nu_l$, $\nu_l^\star = l$, $q_u^\star = q_d$, and $q_d^\star = q_u$), T^3_f is the third component of the weak isospin, and $g_{s,f}$ is the QCD charge for quarks and zero for leptons.

The fermion anomalous dimensions can be calculated using diagrams of Fig. 2:

$$\gamma_{f_R} = \frac{1}{16\pi^2}\left[g^2 Q_f^2 \tan^2\theta_W + \frac{4}{3} g^2_{s,f} + 2\left(G^2_{f,1} + G^2_{f,2}\right)\right.$$

$$+O\left(\frac{m_W^2}{m_{Z'}^2}\right)\Bigg],$$

$$\gamma_{f_L} = \frac{1}{16\pi^2}\left[g^2\left(Q_f^2 - |Q_f|\right)\tan^2\theta_W + \frac{4}{3}g_{s,f}^2 + \frac{g^2}{2}\right.$$
$$+\frac{g^2}{4\cos^2\theta_W} + G_{f,1}^2 + G_{f,2}^2 + G_{f^\star,1}^2 + G_{f^\star,2}^2$$
$$\left.+O\left(\frac{m_W^2}{m_{Z'}^2}\right)\right]. \qquad (21)$$

The RG relations (18) lead to the equations for the parameters $\tilde{Y}_{L,f}$, $\tilde{Y}_{R,f}$, $\tilde{Y}_{\phi_i,1}$, and $\tilde{Y}_{\phi_i,2}$ in the lower order in $m_W^2/m_{Z'}^2$:

$$4\pi^2\tilde{Y}_{R,f}\left(\frac{\beta_{\tilde{g}}^{(1)}}{\tilde{g}^2} + \gamma_{m_{Z'}^2}^{(1)}\right) =$$
$$-G_{f,1}^2\left[2T_f^3\left(\tilde{Y}_{\phi,2} + \tilde{Y}_{\phi_1,1}\right) + \tilde{Y}_{L,f} + \tilde{Y}_{L,f^\star} - 2\tilde{Y}_{R,f}\right]$$
$$-G_{f,2}^2\left[2T_f^3\left(\tilde{Y}_{\phi,2} + \tilde{Y}_{\phi_2,1}\right) + \tilde{Y}_{L,f} + \tilde{Y}_{L,f^\star} - 2\tilde{Y}_{R,f}\right],$$
$$4\pi^2\tilde{Y}_{L,f}\left(\frac{\beta_{\tilde{g}}^{(1)}}{\tilde{g}^2} + \gamma_{m_{Z'}^2}^{(1)}\right) = \frac{g^2}{2}\left(\tilde{Y}_{L,f} - \tilde{Y}_{L,f^\star}\right)$$
$$+\left(G_{f,1}^2 + G_{f,2}^2\right)\left(2T_f^3\tilde{Y}_{\phi,2} + \tilde{Y}_{L,f} - \tilde{Y}_{R,f}\right)$$
$$-G_{f^\star,1}^2\left(2T_f^3\tilde{Y}_{\phi_1,1} - \tilde{Y}_{L,f} + \tilde{Y}_{R,f^\star}\right)$$
$$-G_{f^\star,2}^2\left(2T_f^3\tilde{Y}_{\phi_2,1} - \tilde{Y}_{L,f} + \tilde{Y}_{R,f^\star}\right). \qquad (22)$$

One has to derive two sets of relations which ensure the compatibility of Eqs. (22). The first one is

$$\tilde{Y}_{\phi_2,1} = \tilde{Y}_{\phi_1,1} = -\tilde{Y}_{\phi,2} \equiv -\tilde{Y}_\phi,$$
$$\tilde{Y}_{L,f} + \tilde{Y}_{L,f^\star} = 0, \quad \tilde{Y}_{R,f} = 0. \qquad (23)$$

It describes the Z' analogous to the third component of the $\mathrm{SU}(2)_L$ gauge field. The characteristic features of these interactions are zero traces of the generators and absence of couplings to right-handed singlets. In what follows we will call this type of interaction the "chiral" Z'. The second set,

$$\tilde{Y}_{\phi_1,1} = \tilde{Y}_{\phi_2,1} = \tilde{Y}_{\phi,2} \equiv \tilde{Y}_\phi,$$
$$\tilde{Y}_{L,f} = \tilde{Y}_{L,f^\star}, \quad \tilde{Y}_{R,f} = \tilde{Y}_{L,f} + 2T_f^3\tilde{Y}_\phi, \qquad (24)$$

corresponds to the Abelian Z' boson. In this case the SM Lagrangian appears to be invariant under the $\tilde{\mathrm{U}}(1)$ group associated with the Z'. The first and the second relations in Eqs. (24) mean that the appropriate generators are proportional to the unit matrix, whereas the third relation ensures the Yukawa terms to be invariant with respect to the $\tilde{\mathrm{U}}(1)$ transformations. Introducing the Z' couplings to the vector and the axial-vector fermion currents, $v_{Z'}^f \equiv (\tilde{Y}_{L,f} + \tilde{Y}_{R,f})/2$, $a_{Z'}^f \equiv (\tilde{Y}_{R,f} - \tilde{Y}_{L,f})/2$, one can rewrite the second and the third of Eqs. (24) in the following form:

$$v_{Z'}^f - a_{Z'}^f = v_{Z'}^{f^\star} - a_{Z'}^{f^\star}, \quad a_{Z'}^f = T_f^3\tilde{Y}_\phi. \qquad (25)$$

As is seen, the couplings of the Abelian Z' to the axial-vector fermion currents have the universal absolute value proportional to the Z' coupling to the scalar doublets. The solutions derived are the same as in the case of the minimal SM considered in Ref. [7]. Notice that both of correlations (23) and (24) lead to the same Z' couplings to each of the scalar doublets.

IV. RG CORRELATIONS AND THE Z' IN E_6 BASED MODELS

Over the last decades, the GUT's based on the E_6 gauge group [15] are intensively studied. They predict the Abelian Z' boson with the mass $m_{Z'} \gg m_W$. Since the low-energy limit of the E_6 GUT's is the THDM considered, it is interesting to check whether the relations (25) hold for the specific values of the Z' couplings in these models.

There are different schemes of the E_6-symmetry breaking. One of them is

$$\mathrm{E}_6 \to \mathrm{SO}(10) \times \mathrm{U}(1)_\psi,$$
$$\mathrm{SO}(10) \to \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times$$
$$\times \mathrm{U}(1)_{B-L}. \qquad (26)$$

It leads to the so called left-right (LR) model. Another scheme,

$$\mathrm{E}_6 \to \mathrm{SO}(10) \times \mathrm{U}(1)_\psi \to \mathrm{SU}(5) \times \mathrm{U}(1)_\chi \times \mathrm{U}(1)_\psi, \quad (27)$$

predicts the Abelian Z' which is the linear combination of the neutral vector bosons ψ and χ:

$$Z' = \chi\cos\tilde{\beta} + \psi\sin\tilde{\beta}, \qquad (28)$$

where $\tilde{\beta}$ is the mixing angle.

In Table I (see Ref. [1]) we show the Z' couplings to the SM fermions in the models mentioned (notice, the sign of the axial-vector couplings in Ref. [1] is opposite to the sign of $a_{Z'}^f$). At first glance, some of the couplings in Table I are inconsistent with the relations (25). This requires to be discussed in more detail.

First of all, let us consider the Z' couplings to neutrinos. It is usually supposed in theories based on the E_6 group that the Yukawa terms responsible for generation of the Dirac masses of neutrinos must be set to zero [15]. Therefore, there are no RG relations for the Z' interactions with the neutrino axial-vector currents, since the terms proportional to $G_{\nu,i}$ vanish in Eq. (22). In this case the couplings $a_{Z'}^\nu$ given in Table I are not restricted by the relations (25).

Now, let us discuss the Z' couplings to charged leptons and quarks. The values of the couplings satisfy the relations (25) in the case of the LR model. As for models described by the E_6 breaking scheme (27), two possibilities of choosing $\tilde{\beta}$ are of interest. First is if the ψ boson is much heavier than the χ field. In general, this is a natural condition, since the fields ψ and χ arise at different energy scales. As a consequence, the field ψ is decoupled, and the mixing angle $\tilde{\beta}$ is small ($\tilde{\beta} \ll 1$). In this case the RG relations (25) hold in lower order in $\tilde{\beta}$ for the Z' couplings to quarks and charged leptons.

The second possibility is if the masses of χ and ψ are of the same order. It means the tuning of the vacuum expectation values generating the vector boson masses. This case cannot be treated straightforwardly on the basis of the relations (25), since the mixed states of the Z' bosons have to be included into consideration explicitly. Although our approach is applicable in this case, it requires additional investigation which is not a present subject. Moreover, the Z' mixed states cause the different exchange amplitudes which have to be incorporated into low-energy observables. In what follows the case of two Z' bosons with the masses of the same order will not be discussed.

V. OBSERVABLES

Now, let us introduce the observables convenient for detection of the Z' in electron-positron annihilation into fermion pairs $e^+e^- \to V^* \to \bar{f}f$ $(f \neq e, t)$. The center-of-mass energy is taken in the range $\sqrt{s} \geq 500$ GeV. Consider the case of non-polarized initial and final fermions. Since the t quark is not considered, the fermions can be treated as massless particles, $m_f \sim 0$. In this approximation the left-handed and the right-handed fermions can be substituted by the helicity states, which will be marked as λ and ξ for the incoming electron and the outgoing fermion, respectively $(\lambda, \xi = L, R)$.

Let $\mathcal{A}_V$ denotes the Born amplitude of the process $e^+e^- \to V^* \to \bar{f}f$ $(f \neq e, t)$ with the virtual V-boson state in the s channel $(V = A, Z, Z')$. The Z' boson existence leads to the deviations $\sim m_{Z'}^{-2}$ of the cross section from its SM value. In general, tree-level deviations originate from two types of contributions. The first is caused by the Z-Z' mixing. Using the results of Sec. III, the mixing angle θ_0 [see Eq. (9)] can be calculated as follows,

$$\theta_0 \simeq \frac{\tilde{g} m_W^2 \tilde{Y}_\phi}{g \cos\theta_W m_{Z'}^2}. \tag{29}$$

Because of the mixing there are corrections of order $\theta_0 \sim m_{Z'}^{-2}$ to the vertex, describing interaction of Z boson and fermions, and the amplitude $\mathcal{A}_Z(\theta_0)$ deviates from its SM value $\mathcal{A}_Z(\theta_0 = 0)$. The second type describes the interference between the SM amplitude, $\mathcal{A}_{\mathrm{SM}}$, and the Z' exchange amplitude, $\mathcal{A}_{Z'}$. Thus, the deviation of the cross section of the process $e^+e^- \to \bar{f}f$ is

$$\Delta\frac{d\sigma_f}{d\Omega} = \frac{d\sigma_f}{d\Omega} - \frac{d\sigma_{f,\mathrm{SM}}}{d\Omega} = \frac{\mathrm{Re}\,[\mathcal{A}_{\mathrm{SM}}^* \Delta\mathcal{A}]}{32\pi s} + O\left(\frac{s^2}{m_{Z'}^4}\right), \tag{30}$$

where

$$\mathcal{A}_{\mathrm{SM}} = \mathcal{A}_A + \mathcal{A}_Z|_{\theta_0=0}\,, \quad \Delta\mathcal{A} = \mathcal{A}_{Z'} + \left(\frac{d\mathcal{A}_Z}{d\theta_0}\right)_{\theta_0=0}\theta_0. \tag{31}$$

The quantity $\Delta d\sigma/d\Omega$ can be calculated in the form:

$$\Delta\frac{d\sigma_f}{d\Omega} = \sum_{\lambda,\xi=L,R}\left[\mathcal{I}_{\lambda\xi}^{ef}(s) + \mathcal{M}_{\lambda\xi}^{ef}(s)\right](z + P_\lambda P_\xi)^2, \tag{32}$$

where $P_L = -1$, $P_R = 1$, $z \equiv \cos\theta$ (θ is the angle between the incoming electron and the outgoing fermion), $\mathcal{I}_{\lambda\xi}^{ef}$ denotes the Z-Z' interference term, and $\mathcal{M}_{\lambda\xi}^{ef}$ means the contributions from the Z-Z' mixing:

$$\begin{aligned}
\mathcal{I}_{\lambda\xi}^{ef} &= \frac{\alpha_{\mathrm{em}}\tilde{g}^2 T_f^3 N_f}{4\pi m_{Z'}^2}\tilde{Y}_{\lambda,e}\tilde{Y}_{\xi,f}\,[|Q_f| \\
&\quad + \chi(s)\,(P_\lambda - \varepsilon)\,(P_\xi - 1 + |Q_f| - |Q_f|\varepsilon)]\,, \\
\mathcal{M}_{\lambda\xi}^{ef} &= \frac{\alpha_{\mathrm{em}} g\tilde{g} T_f^3 N_f \theta_0}{4\pi\cos\theta_W (s - m_Z^2)}\left[\tilde{Y}_{\xi,f}\left(\delta_{\lambda,L} - 2\sin^2\theta_W\right)\right. \\
&\quad \left. + 2T_f^3 \tilde{Y}_{\lambda,e}\left(2|Q_f|\sin^2\theta_W - \delta_{\xi,L}\right)\right][|Q_f| \\
&\quad + \chi(s)\,(P_\lambda - \varepsilon)\,(P_\xi - 1 + |Q_f| - |Q_f|\varepsilon)]\,,
\end{aligned} \tag{33}$$

where α_{em} is the fine structure constant, $N_f = 3$ for quarks and $N_f = 1$ for leptons, $\varepsilon \equiv 1 - 4\sin^2\theta_W \sim 0.08$, $\chi^{-1}(s) = 16\sin^2\theta_W\cos^2\theta_W(1 - m_Z^2/s)$, and $\delta_{\lambda,\xi}$ is the Kronecker symbol. The leading contribution originates from the Z-Z' interference term $\mathcal{I}_{\lambda\xi}^{ef}$, whereas the contributions from the Z-Z' mixing $\mathcal{M}_{\lambda\xi}^{ef}$ are suppressed by the additional factor m_Z^2/s. At energies $\sqrt{s} \geq 500$ GeV $\mathcal{M}_{\lambda\xi}^{ef} \ll \mathcal{I}_{\lambda\xi}^{ef}$.

To take into consideration the correlations (23) or (24) let us introduce the function $\sigma(z)$ defined as the difference of cross sections integrated in a suitable range of $\cos\theta$ [8]:

$$\sigma(z) \equiv \int_z^1 \frac{d\sigma}{dz}dz - \int_{-1}^z \frac{d\sigma}{dz}dz. \tag{34}$$

The conventionally used observables – the total cross section σ_T and the forward-backward asymmetry A_{FB} – can be obtained by the special choice of z [$\sigma_T = \sigma(-1)$, $A_{FB} = \sigma(0)/\sigma_T$]. One can express $\sigma(z)$ in terms of σ_T and A_{FB}:

$$\sigma(z) = \sigma_T\left[A_{FB}\left(1 - z^2\right) - \frac{1}{4}z\left(3 + z^2\right)\right]. \tag{35}$$

Then, the deviation $\Delta\sigma(z) \equiv \sigma(z) - \sigma_{\mathrm{SM}}(z)$ can be written in the form:

$$\begin{aligned}
\Delta\sigma_f(z) &= 4\pi\sum_{\lambda,\xi}\left[\mathcal{I}_{\lambda\xi}^{ef}(s) + \mathcal{M}_{\lambda\xi}^{ef}(s)\right] \\
&\quad \times \left(P_\lambda P_\xi - z - z^2 P_\lambda P_\xi - \frac{z^3}{3}\right).
\end{aligned} \tag{36}$$

Let us compare the observable $\Delta\sigma_f(z)$ with the differential cross section (32). As is seen, the polynomial in the polar angle z in Eq. (32) is replaced by the function of the boundary angle z in Eq. (36). The overall factor 4π appears due to the angular integration.

In what follows we consider the observable (36) taking into account the correlations (23) and (24).

A. Chiral Z'

The case of the chiral Z' corresponds to the correlations (23). Because of absence of the Z' couplings to the right-handed fermions the leading contribution to $\Delta\sigma_f(z)$ is proportional to the same polynomial in z for any outgoing fermion f:

$$\begin{aligned}\Delta\sigma_f(z) &\simeq 4\pi\mathcal{I}^{ef}_{LL}(s)\left(1-z-z^2-\frac{z^3}{3}\right)\\ &= \frac{\alpha_{\rm em}\tilde{g}^2T^3_fN_f}{m^2_{Z'}}\tilde{Y}_{L,e}\tilde{Y}_{L,f}\left(1-z-z^2-\frac{z^3}{3}\right)\\ &\quad\times\left\{[|Q_f|+2\chi(s)-|Q_f|\chi(s)]+O\left(\varepsilon\right)\right\}. \quad (37)\end{aligned}$$

Therefore, the differential cross section is completely determined by the total one:

$$\begin{aligned}\Delta\sigma_f(z) = \Delta\sigma_{f,T}\Big[&\frac{3}{4}\left(1-z-z^2-\frac{z^3}{3}\right)\\ &+O\left(\varepsilon, m^2_Z s^{-1}\right)\Big]. \quad (38)\end{aligned}$$

Comparing the observables for fermions of the same $\mathrm{SU}(2)_L$ isodoublet, $\{f_u, f_d\}$, it is possible to derive the correlation:

$$\Delta\sigma_{f_u}(z) = \Delta\sigma_{f_d}(z)\left[\frac{|Q_{f_u}|+1}{|Q_{f_d}|+1}+O\left(\varepsilon, m^2_Z s^{-1}\right)\right]. \quad (39)$$

Hence, the ratio $\Delta\sigma_{f_u}(z)/\Delta\sigma_{f_d}(z)$ is independent of z. It equals to 5/4 for quarks and 1/2 for leptons in lower order in ε, $m^2_Z s^{-1}$. So, the values of the observables in the $\Delta\sigma_{f_u}(z)$ – $\Delta\sigma_{f_d}(z)$ plane are at the same curve (straight line in the approximation used) for any z specified.

It also follows from Eq. (37) that there is the value $z = z'$ when $\Delta\sigma(z') = 0$. As one can check, $z' = 2^{2/3}-1$. Notice, the observable $\Delta\sigma(z')$ is just the variable $\Delta\sigma_-$ proposed in Ref. [16]. This quantity is completely insensitive to the chiral Z' signals. On the other hand, the deviation of the total cross section, $\Delta\sigma_T$, is most sensitive to the signal of the chiral Z', since the maximum of the polynomial $1-z-z^2-z^3/3$ is at $z=-1$.

B. Abelian Z'

The case of the Abelian Z' beyond the minimal SM was considered recently in Ref. [8] where sign-definite observables proper for detection of the Abelian Z' were proposed. The RG correlations (24) in Sec. III coincide with that of Ref. [8]. Therefore, the observables for Abelian Z' beyond the THDM are to be the same as in the case of the minimal SM.

In the case of the chiral Z' the RG correlations (23) suppress the amplitudes corresponding to processes with right-handed fermions. It is not the case for the Abelian Z'. However, one can switch off some contributions to the observable (36) by special choice of the kinematic parameter z. In what follows it is convenient to use the Z' couplings to vector and axial-vector fermion currents $[v^f_{Z'} \equiv (\tilde{Y}_{L,f}+\tilde{Y}_{R,f})/2,\ a^f_{Z'} \equiv (\tilde{Y}_{R,f}-\tilde{Y}_{L,f})/2]$. Because of the correlations (25) the absolute value of the axial-vector couplings is universal for all types of the SM fermions, $a_{Z'} \sim \tilde{Y}_\phi$. So, the observable $\Delta\sigma_f(z)$ depends on the following products of the Z' couplings:

$$\begin{aligned}\Delta\sigma_f(z) = \frac{\alpha_{\rm em}\tilde{g}^2}{m^2_{Z'}}\Big[&\mathcal{F}^f_0(z,s)a^2_{Z'}+\mathcal{F}^f_1(z,s)v^e_{Z'}v^f_{Z'}\\ &+\mathcal{F}^f_2(z,s)a_{Z'}v^f_{Z'}+\mathcal{F}^f_3(z,s)v^e_{Z'}a_{Z'}\Big]. \quad (40)\end{aligned}$$

As it was shown in Ref. [8], it is possible to choose the value of $z = z^*$ which switches off the leading contributions to the leptonic factors $\mathcal{F}^l_1$, $\mathcal{F}^l_2$, and the factor $\mathcal{F}^f_3$. The appropriate value of z^* can be found from the equation:

$$\chi(s)\left(1-z^{*2}\right)-\left(z^*+\frac{z^{*3}}{3}\right)\left[1+\chi(s)\varepsilon^2\right] = 0. \quad (41)$$

The solution $z^*(s)$ is shown in Fig. 3. It switches off the factor at $v^e_{Z'}v^l_{Z'}$. As is seen, z^* decreases from 0.317 at $\sqrt{s} = 500$ GeV to 0.313 at $\sqrt{s} = 700$ GeV. In what follows the value of $\sqrt{s}$ is taken to be 500 GeV, since z^* and $\Delta\sigma(z)$ depend on the center-of-mass energy through the small quantity m^2_Z/s (in fact, the order of such contributions is about 3%).

With the above discussed choice of z^* made, one can introduce the sign definite observable $\Delta\sigma_l(z^*)$:

$$\begin{aligned}\Delta\sigma_l(z^*) &= \frac{\alpha_{\rm em}\tilde{g}^2}{m^2_{Z'}}\mathcal{F}^l_0(z^*,s)a^2_{Z'}\\ &= -0.10\frac{\alpha_{\rm em}\tilde{g}^2\tilde{Y}^2_\phi}{m^2_{Z'}}\left[1+O\left(0.04\right)\right] < 0. \quad (42)\end{aligned}$$

Notice that the value of $\Delta\sigma_l(z^*)$ is universal for all the types of the SM charged leptons. There are also sign definite observables for quarks of the same generation:

$$\Delta\sigma_q(z^*) \equiv \Delta\sigma_{q_u}+0.5\Delta\sigma_{q_d} \simeq 2.45\Delta\sigma_l\left(z^*\right) < 0. \quad (43)$$

Hence one can conclude that the values of $\Delta\sigma_{q_u}(z^*)$ and $\Delta\sigma_{q_d}(z^*)$ in the $\Delta\sigma_{q_u}(z^*)$ – $\Delta\sigma_{q_d}(z^*)$ plane have to be at the line crossing axes at the points $\Delta\sigma_{q_u}(z^*) = 2.45\Delta\sigma_l(z^*)$ and $\Delta\sigma_{q_d}(z^*) = 4.9\Delta\sigma_l(z^*)$, respectively.

Signals of the Abelian and the chiral Z' are compared in Figs. 4-5. Suppose for a moment that experiments give non-zero values of leptonic observables

$\Delta\sigma_l(z^*)$ $(l = \mu, \tau)$. If they correspond to the Abelian Z', both the values are to be the same negative quantity. Let one also know the values of neutrino observables $\Delta\sigma_\nu(z^*)$ $(\nu = \nu_\mu, \nu_\tau)$. In case of the chiral Z' the appropriate point in Fig. 4 has to be at the straight line shown (with the accuracy of the approximation). For the Abelian Z' all shaded region is available. Now, let us consider observables for quarks of the same generation (see Fig. 5). If the value of the leptonic observable $\Delta\sigma_l(z^*)$ is measured, one has to expect that the experimental point in the plot will be at one of two curves corresponding either to the chiral or to the Abelian Z'. The shaded area represents the signals of the Abelian Z' for all possible values of the leptonic observable. So, by measuring the observables $\Delta\sigma_f(z^*)$ for fermions of the same $\mathrm{SU}(2)_L$ isodoublet, one is able to distinguish the Abelian and the chiral Z' couplings.

VI. DISCUSSION

In the present paper the method of the RG relations [7,17], developed originally for the minimal SM, is extended to searching for signals of the heavy Z' gauge boson beyond the THDM. General conditions when our consideration is applicable are the following. 1) The mechanism of generation of the heavy particle masses is not specified and the Z' mass is considered as an arbitrary parameter. 2) The masses of light particles are generated in a standard way via the non-zero vacuum values of the scalar fields of the low-energy basis theory. Interactions of these particles with heavy scalar fields which are responsible for $m_{Z'}$ are excluded at tree level. The radiation corrections to the masses due to loops of heavy particles are suppressed by factors $\sim O(m_{\mathrm{light}}/m_{Z'})$ and therefore not taken into account. This kind of the mass hierarchy corresponds to the case when the basis theory is a subgroup of the underlying high energy one which remains unknown.

As the carried out consideration shown, only two types of the Z' couplings to light particles are consistent with renormalizability. The first type corresponds to the Abelian couplings respecting the $\tilde{\mathrm{U}}(1)$ symmetry of the effective Lagrangian. In this case, the RG correlations fix the gauge symmetry of the Yukawa terms which relates the fermion and the scalar hypercharges. As a consequence, the Z' couplings to the axial-vector fermion currents are completely determined by the scalar field hypercharge and the fermion isospin. The second set of solutions – chiral Z' – describes interactions with the SM particles similar to the third component of the $\mathrm{SU}(2)_L$ gauge field. The characteristic feature of the latter couplings is zero traces of the generators associated with the Z'. Notice that the Z' interactions of the chiral type result in the effective four-fermion couplings $(\bar{f}_{1L}\gamma^\mu\sigma^a f_{1L})(\bar{f}_{2L}\gamma^\mu\sigma^a f_{2L})$ which are described by the operators $\mathcal{O}^{(3)}_{ll}$, $\mathcal{O}^{(3)}_{lq}$, and $\mathcal{O}^{(1,3)}_{qq}$ in accordance with the classification in Refs. [18]. Since each kind of the Z' interactions corresponds to the different operators mentioned, there is a possibility to select them by constructing the proper observables. As was shown, the observables proposed in Ref. [8] can be chosen in searching for the Abelian Z' boson. Thus, the bounds on the Z' couplings calculated therein are also applicable in the case of the THDM.

The above note is important for the model independent search for Z' virtual states at LEP2 and future colliders LHC and NLC. In the analysis of experimental data no discriminations between these two cases have been discussed in literature (see, for instance, recent survey [1] or report [3]). This difference has also to be important for the model-dependent Z' search when different scenarios of the symmetry breaking are discussed.

The derived RG relations have to be useful in improving of experimental bounds on either the parameters of the Z' interaction with fermions and on the relations between the cross sections of various four-fermion scattering processes.

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APPENDIX A: FEYNMAN RULES

In what follows we use the notation $\omega_{L,R} = (1 \mp \gamma^5)/2$, and all the momenta in the vertices are understood to be incoming. The Feynman rules for vertices of Figs. 1, 2 are listed below:

1. Fermion-vector vertices

$$
\begin{aligned}
\bar{f} f A_\mu :&\quad g \sin\theta_W Q_f \gamma^\mu; \\
\bar{f} f Z_\mu :&\quad \frac{g}{\cos\theta_W}\gamma^\mu\left(T^3_f\omega_L - Q_f\sin^2\theta_W\right) \\
&\quad +O(\theta_0); \\
\bar{f} f Z'_\mu :&\quad \frac{\tilde{g}}{2}\gamma^\mu\left(\omega_L\tilde{Y}_{L,f} + \omega_R\tilde{Y}_{R,f}\right) + O(\theta_0); \\
\bar{f}_d f_u W^-_\mu :&\quad \frac{g}{\sqrt{2}}\gamma^\mu\omega_L; \\
\bar{f}_u f_d W^+_\mu :&\quad \frac{g}{\sqrt{2}}\gamma^\mu\omega_L;
\end{aligned}
$$

2. Fermion-scalar vertices

$$
\begin{aligned}
\bar{f} f H :&\quad -\left(G_{f,1}\cos\alpha + G_{f,2}\sin\alpha\right); \\
\bar{f} f h :&\quad \left(G_{f,1}\sin\alpha - G_{f,2}\cos\alpha\right); \\
\bar{f} f A_0 :&\quad 2iT^3_f\left(\omega_L - \omega_R\right) \\
&\quad \times\left(G_{f,1}\sin\beta - G_{f,2}\cos\beta\right); \\
\bar{f} f \chi_3 :&\quad -2iT^3_f\left(\omega_L - \omega_R\right)
\end{aligned}
$$

$$\times \left(G_{f,1}\cos\beta + G_{f,2}\sin\beta\right);$$

$$\bar{f}_d f_u H^- : \quad \sqrt{2}\left[\omega_L\left(G_{f_d,1}\sin\beta - G_{f_d,2}\cos\beta\right) + \omega_R\left(-G_{f_u,1}\sin\beta + G_{f_u,2}\cos\beta\right)\right];$$

$$\bar{f}_u f_d H^+ : \quad \sqrt{2}\left[\omega_R\left(G_{f_d,1}\sin\beta - G_{f_d,2}\cos\beta\right) + \omega_L\left(-G_{f_u,1}\sin\beta + G_{f_u,2}\cos\beta\right)\right];$$

$$\bar{f}_d f_u \chi^- : \quad \sqrt{2}\left[-\omega_L\left(G_{f_d,1}\cos\beta + G_{f_d,2}\sin\beta\right) + \omega_R\left(G_{f_u,1}\cos\beta + G_{f_u,2}\sin\beta\right)\right];$$

$$\bar{f}_u f_d \chi^+ : \quad \sqrt{2}\left[-\omega_R\left(G_{f_d,1}\cos\beta + G_{f_d,2}\sin\beta\right) + \omega_L\left(G_{f_u,1}\cos\beta + G_{f_u,2}\sin\beta\right)\right];$$

3. Z' scalar vertices

$$Z'_\mu H^+ H^- : \quad \frac{\tilde{g}}{2}\left(p_{H^+} - p_{H^-}\right)_\mu \left(\tilde{Y}_{\phi_1,1}\sin^2\beta + \tilde{Y}_{\phi_2,1}\cos^2\beta\right) + O(\theta_0);$$

$$Z'_\mu H^+ \chi^- : \quad \frac{\tilde{g}\sin 2\beta}{4}\left(p_{\chi^-} - p_{H^+}\right)_\mu \times \left(\tilde{Y}_{\phi_1,1} - \tilde{Y}_{\phi_2,1}\right) + O(\theta_0);$$

$$Z'_\mu H^- \chi^+ : \quad \frac{\tilde{g}\sin 2\beta}{4}\left(p_{H^-} - p_{\chi^+}\right)_\mu \times \left(\tilde{Y}_{\phi_1,1} - \tilde{Y}_{\phi_2,1}\right) + O(\theta_0);$$

$$Z'_\mu \chi^+ \chi^- : \quad \frac{\tilde{g}}{2}\left(p_{\chi^+} - p_{\chi^-}\right)_\mu \left(\tilde{Y}_{\phi_1,1}\cos^2\beta + \tilde{Y}_{\phi_2,1}\sin^2\beta\right) + O(\theta_0);$$

$$Z'_\mu H A_0 : \quad \frac{i\tilde{g}}{2}\left(p_{A_0} - p_H\right)_\mu \tilde{Y}_{\phi,2}\sin(\alpha-\beta) + O(\theta_0);$$

$$Z'_\mu H \chi_3 : \quad \frac{i\tilde{g}}{2}\left(p_{\chi_3} - p_H\right)_\mu \tilde{Y}_{\phi,2}\cos(\alpha-\beta) + O(\theta_0);$$

$$Z'_\mu h A_0 : \quad \frac{i\tilde{g}}{2}\left(p_{A_0} - p_h\right)_\mu \tilde{Y}_{\phi,2}\cos(\alpha-\beta) + O(\theta_0);$$

$$Z'_\mu h \chi_3 : \quad \frac{i\tilde{g}}{2}\left(p_h - p_{\chi_3}\right)_\mu \tilde{Y}_{\phi,2}\sin(\alpha-\beta) + O(\theta_0).$$

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TABLE I. The Z' couplings to the SM fermions in the E_6 and LR models.

f	E_6: $a^f_{Z'}$	$v^f_{Z'}$	LR: $a^f_{Z'}$	$v^f_{Z'}$
ν	$-3\frac{\cos\tilde{\beta}}{\sqrt{40}} - \frac{\sin\tilde{\beta}}{\sqrt{24}}$	$3\frac{\cos\tilde{\beta}}{\sqrt{40}} + \frac{\sin\tilde{\beta}}{\sqrt{24}}$	$-\frac{1}{2\alpha}$	$\frac{1}{2\alpha}$
e	$-\frac{\cos\tilde{\beta}}{\sqrt{10}} - \frac{\sin\tilde{\beta}}{\sqrt{6}}$	$2\frac{\cos\tilde{\beta}}{\sqrt{10}}$	$-\frac{\alpha}{2}$	$\frac{1}{\alpha} - \frac{\alpha}{2}$
u	$\frac{\cos\tilde{\beta}}{\sqrt{10}} - \frac{\sin\tilde{\beta}}{\sqrt{6}}$	0	$\frac{\alpha}{2}$	$-\frac{1}{3\alpha} + \frac{\alpha}{2}$
d	$-\frac{\cos\tilde{\beta}}{\sqrt{10}} - \frac{\sin\tilde{\beta}}{\sqrt{6}}$	$-2\frac{\cos\tilde{\beta}}{\sqrt{10}}$	$-\frac{\alpha}{2}$	$-\frac{1}{3\alpha} - \frac{\alpha}{2}$

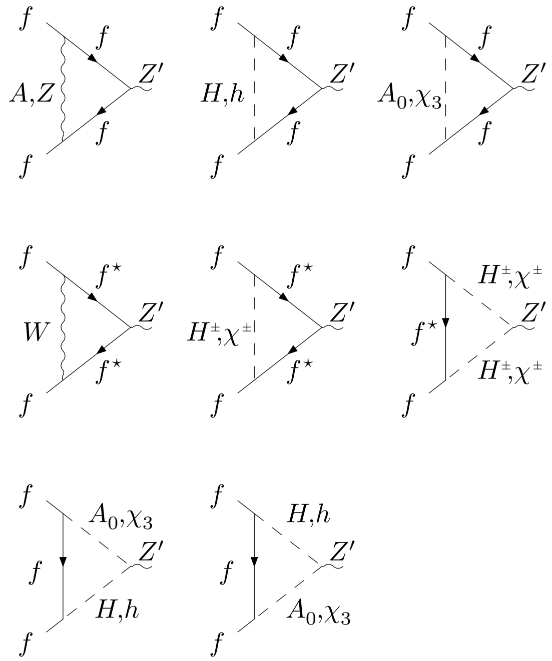

FIG. 1. One-loop contributions to the divergent part of $\Gamma_{fZ'}$.

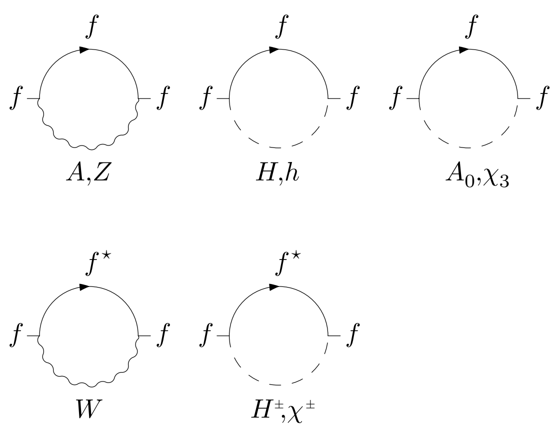

FIG. 2. One-loop contributions to the fermion mass operator.

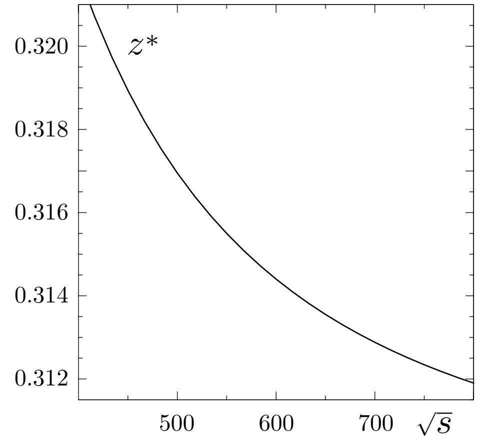

FIG. 3. z^* as the function of $\sqrt{s}$(GeV).

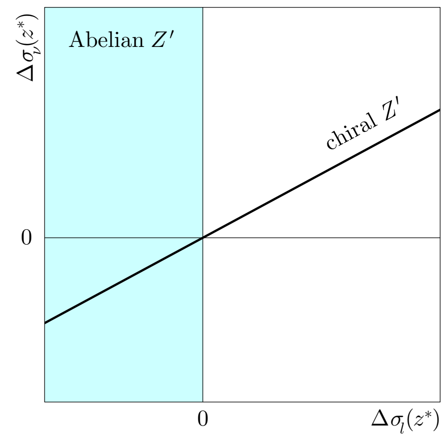

FIG. 4. Signals of the Abelian and the chiral Z' in the plane of observables $\Delta\sigma_{q_l}(z^*)$ and $\Delta\sigma_{q_{\nu_l}}(z^*)$ for leptons of the same generation. The shaded area represents the signal of the Abelian Z' for all possible values of the axial-vector couplings $a^f_{Z'}$.

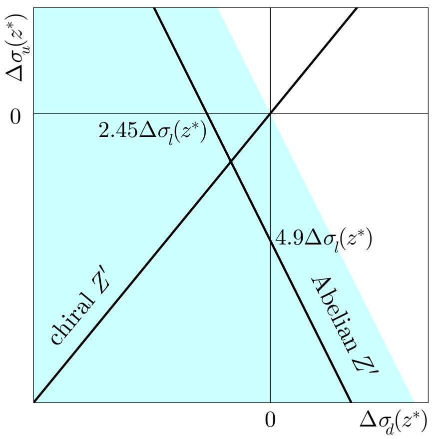

FIG. 5. Signals of the Abelian and the chiral Z' in the plane of observables $\Delta\sigma_{q_d}(z^*)$ and $\Delta\sigma_{q_u}(z^*)$ for quarks of the same generation. The shaded area represents the signal of the Abelian Z' for all possible values of the axial-vector couplings $a^f_{Z'}$.